Addition and Subtraction Game Complementary Module

Contents

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Project Title: Learning to Solve Subtraction and Addition Word Problems Through a Computer-Based Interactive Game

Project Team:

Principal Investigator: Dr. Evrim Erbilgin

Co-Investigators: Dr. Jason Johnson; Dr. Patricia Fidalgo; Dr. Farah El Zein; Dr. Serigne Gningue; Dr. Adeeb Jarrah; Dr. Jennifer Robinson; Dr. Rehab AlHakmani; Dr. Carla Amoi Rudder.

كلية الإمارات للتطوير التربوي
Emirates College for Advanced Education

Theoretical Underpinnings

Teaching how to solve arithmetic word problems has long been an interest of mathematics educators due to the challenges that students face when working with word problems. Several models have been suggested to support student's problem-solving processes: *concrete-pictorial-abstract model* and *bar model/tape diagram*.

Concrete-Pictorial-Abstract

The Concrete Pictorial Abstract (CPA) approach is a system of learning that uses connections between different forms and modalities of a concept to help students build meaningful understanding of mathematical ideas (Kaur, 2019). This instructional strategy has been shown to be effective in increasing the understanding of abstract mathematical concepts for students. Typically, first, students engage in mathematical tasks through the use of concrete resources (e.g., blocks). They are then encouraged to create pictorial representations or visual models of the concrete objects they were using. The CPA approach continues with students' use of abstract representations such as writing mathematical equations.

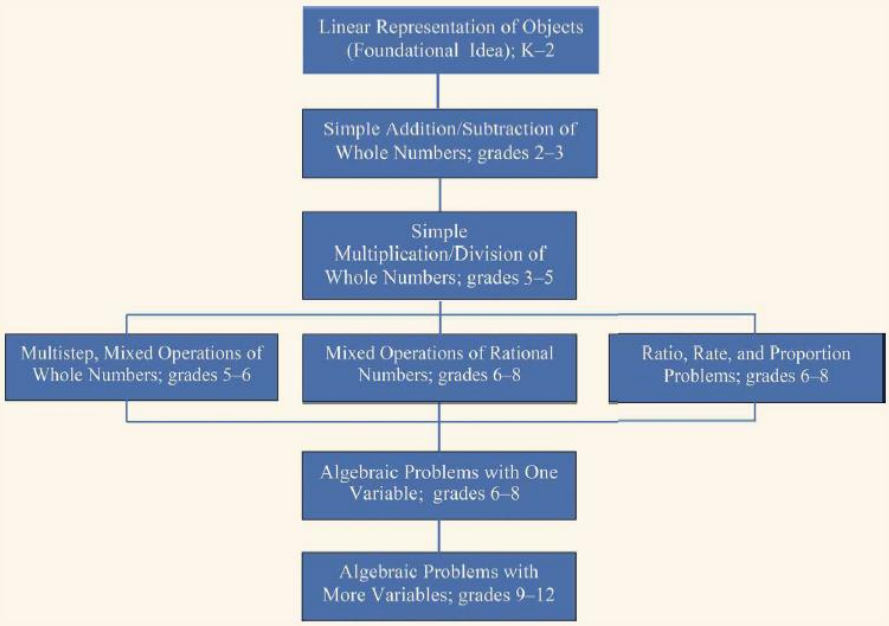
It is very important for students to make connections among these three different representations to build a rich understanding of the concept under investigation.

Bar Model/Tape Diagram

The bar model, also named as tape diagram or strip diagram, has been widely used in elementary grades in many mathematically top performing countries such as Singapore or Japan (Ding, 2018; Murata, 2008). For example, in Japan, bar models are consistently used as the central visual representation across grade levels to enrich the teaching and learning process by focusing on visual illustration of mathematical relationships in word problems. This module extensively utilizes the bar model to promote conceptualization of addition and subtraction situations and problem solving.

In fact, the bar model is not only useful in elementary grade levels. As the figure below shows, it can be used across grade levels from Cycle 1 to Cycle 3 in a variety of mathematical topics. Introducing the bar model in early grade levels will lay the foundation for its use in later grade levels.

Fig. 1



The current module is designed for second grade students. Up to this grade level, students have typically been taught to use concrete objects and draw visual representations of these objects in solving addition or subtraction problems. They are now ready for a conceptual and gradual introduction to the bar model. This module follows a transitive progression from concrete objects to pictorial representations to introduce the bar model. Students are also encouraged to write number sentences, supporting their transition to more abstract thinking. This approach aligns with the Concrete-Pictorial-Abstract (CPA) strategy discussed in the previous section. Figures 2 and 3 illustrate the approach followed in the module. The figures are from the work of She and Harrington (2022).

Fig. 2

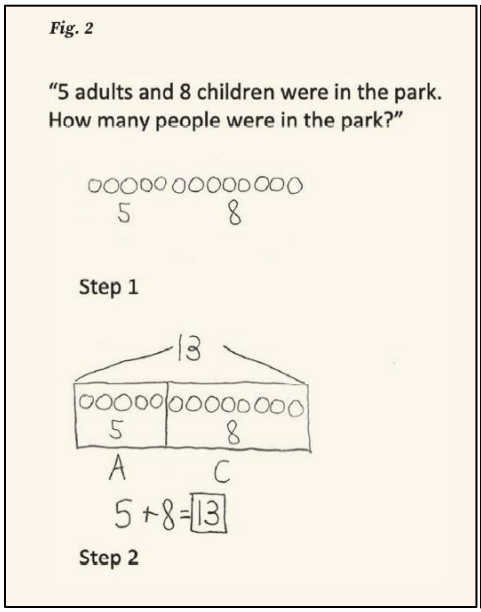
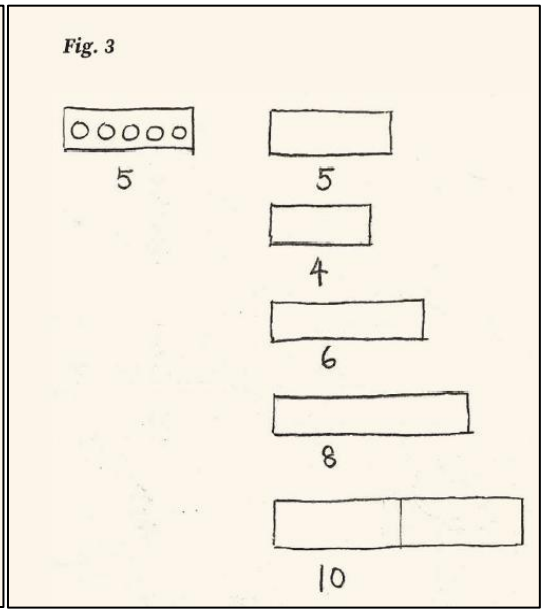


Fig. 3



Addition and Subtraction Problem Structures

Contextual problems are very important teaching tools that we use to help children enhance problem-solving strategies. Researchers have categorized addition and subtraction problems into structures based on the kinds of relationships involved (Verschaffel et al., 2007). Figure 4 summarizes problem structures using number family 4, 8, and 12. The figure is taken from Van De Walle et al. (2020). To help students construct a rich and robust understanding of arithmetic word problems, it is important to use a variety of problem structures in the instructional processes (Carpenter et al., 1988).

Fig. 4.

Problem Type and Structure with Physical Action Involved: Change Problems			
	Result Unknown	**Change Unknown**	**Start Unknown**
Join (add to) **(a)** 	Sandra had 8 pennies. George gave her 4 more. How many pennies does Sandra have altogether? 8 + 4 = □	Sandra had 8 pennies. George gave her some more. Now Sandra has 12 pennies. How many did George give her? 8 + □ = 12	Sandra had some pennies. George gave her 4 more. Now Sandra has 12 pennies. How many pennies did Sandra have to begin with? □ + 4 = 12
Separate (take from) **(b)** 	Sandra had 12 pennies. She gave 4 pennies to George. How many pennies does Sandra have now? 12 − 4 = □	Sandra had 12 pennies. She gave some to George. Now she has 8 pennies. How many did she give to George? 12 − □ = 8	Sandra had some pennies. She gave 4 to George. Now Sandra has 8 pennies left. How many pennies did Sandra have to begin with? □ − 4 = 8
Problem Type and Structure with No Physical Action Involved: Part-Part-Whole and Compare Problems			
	Whole Unknown	One Part Unknown	Both Parts Unknown
Part-Part-Whole **(c)** 	George has 4 pennies and 8 nickels. How many coins does he have? 4 + 8 =	George has 12 coins. Eight of his coins are pennies, and the rest are nickels. How many nickels does George have? 12 = 4 + □ or 12 − 4 = □	George has 12 coins. Some are pennies and some are nickels. How many of each coin could he have? 12 = □ + □
	Difference Unknown	Larger Quantity Unknown	Smaller Quantity Unknown
	Situations of How many more?		
Compare **(d)** 	George has 12 pennies, and Sandra has 8 pennies. How many more pennies does George have than Sandra? 8 + □ = 12	George has 4 more pennies than Sandra. Sandra has 8 pennies. How many pennies does George have? 8 + 4 = □	George has 4 more pennies than Sandra. George has 12 pennies. How many pennies does Sandra have? □ + 4 = 12
	Situations of How many fewer?		
	George has 12 pennies. Sandra has 8 pennies. How many fewer pennies does Sandra have than George? 12 − 8 = □	Sandra has 4 fewer pennies than George. Sandra has 8 pennies. How many pennies does George have? □ − 4 = 8	Sandra has 4 fewer pennies than George. George has 12 pennies. How many pennies does Sandra have? 12 − 4 = □

Lesson Activities Overview

Lesson 1: Introduction to the Bar Model

- Students are introduced to the concept and structure of the bar model.

Lesson 2: Practicing the Bar Model – Part 1

- Solve one-step addition and subtraction problems (join, separate, part-part-whole) with **result unknown** problem situations.

Lesson 3: Game – Level 1

- Play level 1 of the digital game involving addition and subtraction problems (join and separate) with the **result unknown** problem situations.

Lesson 4: Practicing the Bar Model – Part 2

- Solve one-step addition and subtraction problems (join, separate, part-part-whole) with **change or start unknown**.

Lesson 5: Game – Level 2

- Play level 2 of the digital game with problems (join and separate) involving **change or start unknown**.

Lesson 6: Using the Bar Model for Comparison Problems

- Solve one-step **comparison problems** using the bar model.

Lesson 7: Game – Level 3

- Play level 3 of the digital game focusing on **comparison problems**.

Lesson 8: Game – Level 4

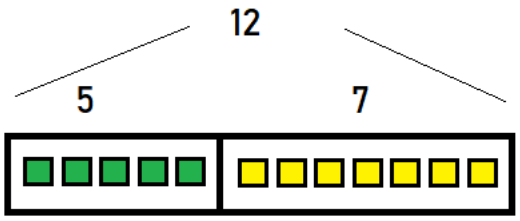
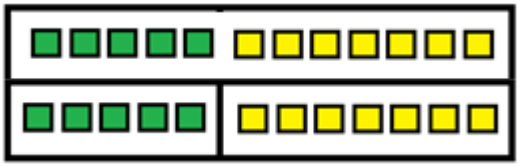
- Play level 4 of the digital game featuring **mixed problem types**.

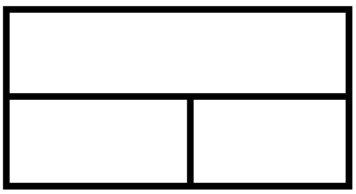
Module Activities

Lesson 1: Introduction to the Bar Model

Grade Level: 2
Time: 1 period
Materials: Color tiles

Learning Objectives:
By the end of this lesson, students should be able to
-draw a bar model to represent the quantities of physical objects when they are joined together or separated from each other.
-discuss the similarities and differences between bar models and physical objects.

Introduction	**Teacher Notes**
Pose the following problem to students. *Aya and Rashed are playing with square tiles. Aya has 5 green tiles. Rashed has 7 yellow tiles. How many tiles do they have altogether?* Share one student's work (preferably with a linear drawing) and draw rectangles around color tiles to introduce the bars:  Also show that the total amount could be represented at the top by using an additional bar: 	Ask metacognitive questions: What is given in the problem? What is being asked? What strategy can you use to solve it? Encourage students to make drawings. Observe students' drawings and use the one with a linear arrangement to introduce drawing a bar around the objects. Tell students that this is called a bar, and it is helpful to organize quantities in a problem. Putting rectangles around corresponding squares helps students visualize the additive relationship of two quantities rather than simply counting discrete objects together. Ask students: What might be the benefits of using bars to solve addition problems?
Body: Playing a Game Students will play the following game in pairs. Give each pair a blank template of a bar model on A4 size paper:	This game will help students to practice the new representation that they have just learned. Illustrate the game with a student. First you need to add the tiles to the A4 game board.

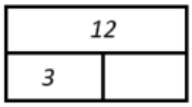


Each student will roll a die and then place that number of tiles to one of the bottom parts of the bar model. Students will work together to calculate the number of tiles that should go to the top part. They will record it on the game sheet.

Instead of a die, students can use spinners or number cards. Also, for more challenge, they can roll two dice to decide the number of tiles that will go on each section instead of rolling one die.

Then, to record each round on the game sheet, ask student if writing the number instead of drawing the objects on the bar model is acceptable. You may refer to the difficulty of drawing large number of objects and ease of just writing the number.

At the end of the game, consider showing them the following bar model and ask what the second person rolled.

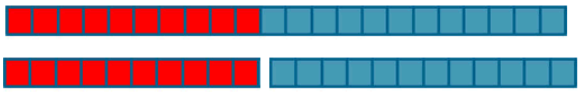


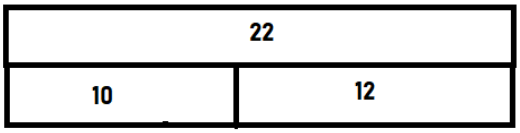
Game sheet:

Die 1	Die 2	Bar model	Number sentence
4	2	6 / 4 / 2	4+2=6

Closure

Show the following image to the students:





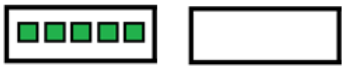
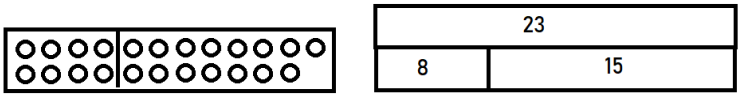
Ask: How are the two images similar? How are they different?

Have different students share their opinions. Highlight that the squares show all the objects physically while the bar model shows the objects numerically. If we have lots of objects, a bar model is easier to draw. The similarity is that they both can show the addition and subtraction situations.

Lesson 2: Practicing the Bar Model – Part 1

Grade Level: 2
Time: 1 period
Materials: Color tiles, base 10 blocks

Learning Objectives:
By the end of this lesson, students should be able to:
-use a bar model to solve one-step addition and subtraction problems (join, separate, part-part-whole) with an unknown result.

Introduction	**Teacher Notes**
Distribute worksheet 1 to students and ask them to draw bars of different sizes given the bar showing 5 objects. **Worksheet 1:** The following bar shows 5 objects.  Draw bars for the following number of objects: A bar to show 3 objects: A bar to show 6 objects: A bar to show 8 objects: A bar to show 10 objects: Have a class discussion and encourage students to share their strategies in drawing the bars.	Worksheet 1 asks students to draw different bars given the size of a bar that shows 5 objects. This practice helps students grasp the idea that quantities can be estimated and represented by bars with varied lengths. You can allow struggling students to use color tiles. Consider asking students to swap their answers with a peer and check each other's work. Strategies may include multiplicative thinking such as doubling the size of the original bar to draw the bar for 10 objects.
Body Pose the following **problem** to students: *Rana had 8 stickers. George gave her 15 more. How many stickers does Rana have altogether?* When sharing the solution to this problem, use the bar model and remind students of the meaning of each part.  Make connections between the drawing and the bar model.	Consider asking metacognitive questions: Who can explain this problem using their own words? What strategy can we use to solve this problem? Explain to students that the bottom parts show the quantities that join, and the top part shows the total amount. Tell students that they will use it to understand and solve addition and subtraction problems.

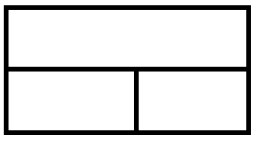
Pose the following **problem** to students and ask them to solve it by drawing a bar model: *Rana had 30 stickers. She gave 12 stickers to George. How many stickers does Rana have now?* Have students share their bar models on the board.	Consider using incorrectly drawn bar models as an opportunity to clarify how to create a bar model.
Allow students to practice using bar models to solve simple addition and subtraction problems. Examples:	Bar models mainly help students understand the relationship between the numbers involved in the problem so they can decide which operation to perform. To perform the operation, students can use base 10 blocks to calculate the answer correctly.
Alice studied mathematics for 25 minutes. Later, she studied for 17 more minutes. How many minutes in total did Alice study?	This problem is a join situation with an unknown result.
Abdallah had 60 fils. He gave 25 fils to his friend. How many fils does Abdallah have now?	This problem is a separate situation with an unknown result.
24 adults and 35 children were in the park. How many people were in the park?	This problem is a Part-part-whole situation with an unknown result.

Closure

Use the following exit ticket to assess students' understanding of bar models and problem situations:

There are 30 adults and children in the park. How many adults could be in the park? How many children could be in the park?

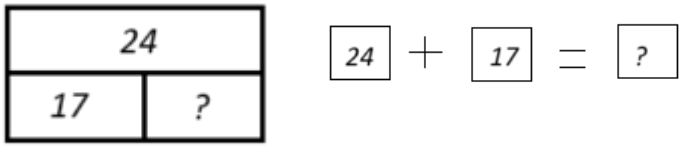
Use the following bar model to solve the problem:



Lesson 3: Game – Level 1

Grade Level: 2
Time: 1 period
Materials: Tablets, base 10 blocks

Learning Objectives:
By the end of this lesson, students should be able to:
-use a bar model to solve one-step addition and subtraction problems (join, separate, part-part-whole) with an unknown result.

Introduction	**Teacher Notes**
Re-visit the exit ticket from lesson 2 to share different solution strategies. In lesson 3, students will play level 1 of the game. Please introduce the game to the students by leading the solution process for a problem.	Support students to notice the relationship between three quantities represented on a bar model. Ask questions such as: How do the two bottom sections relate to the top part?
Body In this lesson, students will play the game using their tablets or laptops individually or in pairs (10-15 minutes).	Make base 10 blocks available to students to support the calculation process.
Closure Use the following exit ticket: Ahmed will solve the following problem: *There are 24 bananas and 17 apples on a tray. How many fruits are there in total?* Ahmed made the following bar model and wrote the number sentence for this problem.  Did Ahmed make a correct bar model for this problem? Is his number sentence correct? Explain your thinking.	Use think-pair-share model to answer this problem. Encourage students to explain their thinking through making a new bar model.

Lesson 4: Practicing the Bar Model – Part 2

Grade Level: 2
Time: 1 period
Materials: Base 10 blocks

Learning Objectives:
By the end of this lesson, students should be able to:
-use a bar model to solve one-step addition and subtraction problems (join, separate, part-part-whole) with an unknown change or start.

Introduction	**Teacher Notes**
Show the following two problems to students. Ask, how do the two problems are different? How are they the same? *Alice ate 15 peanuts in the morning. She ate some more in the afternoon. In total, Alice ate 41 peanuts. How many peanuts did Alice eat in the afternoon?* *Alice ate 15 peanuts in the morning. She ate 26 more in the afternoon. How many peanuts did Alice eat in total?* Have a class discussion on the similarities and differences.	Encourage students to create a **bar model** for each problem. Also encourage writing a **number sentence** for each problem. Draw their attention to differences in problem structure. In one problem, the result is unknown; in the other, the change is unknown. Students need to understand the structure of the problem before deciding which operation to use.
Body Students will solve addition and subtraction problems (join, separate, part-part-whole) with an unknown change or start. Separate with change unknown: *Denis had 45 stickers. He gave some to his friend. Now he has 23 stickers. How many stickers did Denis give to his friend?* Part-part-whole with part unknown: In a bag, there are 59 balls. 35 of these balls are blue, and the remaining balls are yellow. How many yellow balls are there in the bag? Join with start unknown: Omar had some pencils. His brother gave him 16 more pencils. Now, Omar has 38 pencils. How many pencils did Omar have to begin with? Separate with start unknown:	Continue encouraging students to create a **bar model** for each problem. Also encourage writing a **number sentence** for each problem. Make base 10 blocks available to students to support the calculation process.

Alya had some tomatoes in her garden. She picked 12 tomatoes. Then she had 14 tomatoes left. How many tomatoes did Alya have to begin with?	
Closure Use the following exit ticket: Aysha has 68 color tiles. She has 9 blue tiles and 40 red tiles. The rest are green. How many green tiles does Aysha have? Can you draw a bar model to help you figure it out? How many parts will you need at the bottom of your bar model?	Use think-pair-share model to answer this problem.

Lesson 5: Game – Level 2

Grade Level: 2 **Time:** 1 period **Materials:** Tablets, base 10 blocks	
Learning Objectives: By the end of this lesson, students should be able to: -use a bar model to solve one-step addition and subtraction problems (join, separate, part-part-whole) with an unknown change or start.	
Introduction In this lesson, students will play level 2 of the game. The teacher will introduce the level to the students. The class will solve one question together.	**Teacher Notes** Support students to notice the relationship between three different representations: the garden, the bar model, and the number sentence. Ask questions such as: How do the number of fruits in the garden relate to the numbers on the bar model? How do the numbers on the bar model relate to the number sentence on the screen?
Body In this lesson, students will play level 2 of the game using their tablets or laptops individually or in pairs (10-15 minutes).	Make base 10 blocks available to students to support the calculation process.
Closure Use the following exit ticket: Write a word problem that can be solved by the following equation: 	

Lesson 6: Using the Bar Model for Comparison Problems

<table>
<tr><td colspan="2">Grade Level: 2
Time: 1 period</td></tr>
<tr><td colspan="2">Learning Objectives:
By the end of this lesson, students should be able to:
-use a bar model to solve one-step comparison problems.</td></tr>
<tr><td>Introduction

Share sample problems written by students for the exit ticket from the previous day.

Show the following two problems to students. Ask, how do the two problems are different? How are they the same?

Leo rode his bicycle for 32 minutes. Then he rode his bicycle for 17 minutes more. How many minutes did Leo ride his bicycle in total?

Leo rode his bicycle for 32 minutes and Noor rode her bicycle for 17 minutes. How many more minutes did Leo ride than Noor?

Have a class discussion on the similarities and differences.</td><td>Teacher Notes

Discuss if they are correct or incorrect. Invite students to improve those problems.

Encourage students to create a bar model for each problem. Also encourage writing a number sentence for each problem.
Draw their attention to the differences in problem structures. In one problem, the situation is a join situation while in the other, the situation is comparison.</td></tr>
<tr><td>Body

In this lesson, students will practice using bar models to solve comparison problems.

Examples:

Comparison with difference unknown:

Alya studied mathematics for 26 minutes. Zayed studied mathematics for 45 minutes. How many more minutes did Zayed study mathematics than Alya?

Comparison with one quantity unknown:

Sandra has 7 marbles fewer than Mariam. Sandra has 75 marbles. How many marbles does Mariam have?</td><td>

Encourage students to create a bar model for each problem. Also encourage writing a number sentence for each problem.</td></tr>
<tr><td>Closure

Ask students to write a word problem that includes a comparison situation.</td><td>

If time permits, consider asking students to exchange their problems and solve!</td></tr>
</table>

Lesson 7: Game – Level 3

Grade Level: 2 **Time:** 1 period **Materials:** Tablets, base 10 blocks	
Learning Objectives: By the end of this lesson, students should be able to: - use a bar model to solve one-step comparison problems.	
Introduction In this lesson, students will play level 3 of the game. The teacher will introduce the level to the students. The class will solve one question together.	**Teacher Notes** Support students to notice the relationship between three different representations: the garden, the bar model, and the number sentence. Ask questions such as: How do the number of fruits in the garden relate to the numbers on the bar model? How do the numbers on the bar model relate to the number sentence on the screen?
Body In this lesson, students will play level 3 of the game using their tablets or laptops individually or in pairs (10-15 minutes).	Make base 10 blocks available to students to support the calculation process.
Closure Use the following exit ticket: Sandra has 12 tomatoes fewer than Mariam. Sandra has 55 tomatoes. How many tomatoes does Mariam have?	Use think-pair-share model to answer this problem.

Lesson 8: Game – Level 4

Grade Level: 2 **Time:** 1 period **Materials:** Tablets, base 10 blocks	
Learning Objectives: By the end of this lesson, students should be able to: -use a bar model to solve one-step addition and subtraction problems.	
Introduction In this lesson, students will play level 4 of the game. This level produces mixed problem structures from previous levels. The teacher will introduce the level to the students. The class will solve one question together.	**Teacher Notes** Support students to notice the relationship between three different representations: the garden, the bar model, and the number sentence. Ask questions such as:

	How do the number of fruits in the garden relate to the numbers on the bar model? How do the numbers on the bar model relate to the number sentence on the screen?
Body In this lesson, students will play level 4 of the game using their tablets or laptops individually or in pairs (10-15 minutes).	Make base 10 blocks available to students to support the calculation process.
Closure Use the following exit ticket: *David had 13 pencils. He gave some to Anna. Now he has 8 pencils left. How many pencils did David give to Anna?* Which number sentence best represents the situation in the problem? Circle the correct answer. A) 13 − 8 = ? B) 13 − ? = 8 C) 8 + ? = 13	Use think-pair-share model to answer this problem.

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